Ion acoustic waves in plasmas with collisional electrons

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We have developed an analytical theory of ion acoustic wave (IAW) damping and electron heat conductivity in plasmas with an arbitrary electron collisionality. We have found that deviations from the collisionless electron Landau damping occur for the wave numbers $k\lambda_{ei} \sim Z^{2/3}$, where λ_{ei} is the electron mean free path for collisions with ions. In the regime of intermediate collisionality the IAW damping is proportional to $k^{4/7}$ and the nonlocal electron heat conductivity is proportional to $k^{-4/7}$. The transition into the hydrodynamic regime occurs for $k\lambda_{ei} \sim 0.1Z^{1/2}$.

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Ion acoustic waves (IAW) are collective modes which play a fundamental role in the low-frequency response of unmagnetized plasmas. Properties of IAW are well known in both collisional and collisionless limits. Between these two well-known extremes, however, there exists a wide range of intermediate collisionality, which is relevant to fusion plasmas and many other practical applications. Theoretical understanding of IAW in this regime of parameters has to follow from kinetic studies and it is still far from being completed [1].

The regime of intermediate collisionality ($0.01 < k\lambda_{ei} < 100$, where λ_{ei} is the electron-ion (e-i) collision mean free path), which is studied in this paper, can be defined in the long wavelength limit by a discrepancy, which occurs at $k\lambda_{ei} \gtrsim 0.01$, between classical Braginskii [2] theory predictions for the IAW and numerical solutions to the Fokker-Planck equation [1,3]. In the short wavelength limit the transition between collisionless IAW and the regime of weak collisions occurs at $k\lambda_{ei} \lesssim 100$, where the fluidlike description of phase mixing effects by Hammett and Perkins [4] loses its validity.

The contribution from ion-ion (i-i) collisions to IAW dispersion and damping can be accounted for in terms of generalized high-frequency hydrodynamic equations, which have been derived in our previous study [5] using the Grad method. The effect of electron collisions on IAW is far more complex and therefore still poorly understood. In this paper we present analytical results from the kinetic study of IAW damping, which includes e-i and e-e collisions in the entire range of plasma collisionality. We have calculated the disturbance of the electron distribution function due to IAW and have shown the effects of e-e collisions in the region of small particle velocities. Due to these effects the electron Landau damping term becomes modified as soon as $k\lambda_{ei} \sim Z^{2/3}$. The importance of e-e collisions for the electron heat flux is also discussed. Our theory demonstrates that the transition from isothermal to adiabatic electron perturbations takes place in the region $0.1Z^{-1/2} \lesssim k\lambda_{ei} \lesssim Z^{2/3}$ and manifests itself in a $k^{-4/7}$ dependence of the electron heat conductivity coefficient. We have applied our theory to equilibrium plasmas with Maxwellian electron distribution function and obtained a good agreement with Fokker-Planck numerical simulations.

As a reference state we consider a homogeneous plasma with a Maxwellian distribution function for electrons and ions. We assume that the electron temperature is much larger than the ion temperature, $T_e \gg T_i$, and that ions are highly ionized, $Z \gg 1$. Later, we will show how our results can be generalized to ions with arbitrary charge. We also neglect energy exchange between plasma species. Following previous studies [3,6], we consider a small amplitude plane IAW with frequency ω and wave number k. The electron response corresponds to a disturbance of the electron distribution function $f_e(v,\mu) = \Sigma_{l=0}^{\infty} f_l(v)P_l(\mu)$ expanded in a series of the Legendre polynomials $P_l(\mu)$, which are eigenfunctions of the e-i collisional operator, and $\mu = \mathbf{k}\cdot\mathbf{v}/kv$. As a starting point of our kinetic model we take the infinite hierarchy of equations for the harmonics $f_l(v)$ of the electron distribution function as derived in Ref. [1]. We assume a low-frequency perturbation, $\omega \ll kv_{T_e}$ ($v_{T_e} = \sqrt{T_e/m_e}$), and since $Z \gg 1$ we neglect e-e collision terms everywhere as compared to e-i contributions, except in the equation for the symmetric part of the electron distribution f_0. The Z times more frequent e-i collision affect only the anisotropic part of the distribution function and therefore are not present in this equation.

To describe damping of IAW in the collisionless and weakly collisional regime one has to solve an infinite set of equations for higher order angular harmonics of the electron distribution function. This has been accomplished in Refs. [1,7] by introducing a renormalized e-i collision frequency $\tilde{\nu}_1 = \nu_{ei}H_1(kv/\nu_{ei})$, where $\nu_{ei}(v) = 4\pi Z n_e e^4 \Lambda/m_e^2 v^3$ is the velocity-dependent e-i collision rate, n_e is the electron density, and Λ is the Coulomb logarithm. The factor H_1 has been approximated by the simple function $H_1(x) = \sqrt{1+(\pi x/6)^2}$ [1]. After this renormalization equations for the first and second harmonic of the electron distribution function read [1]

$$-i\omega f_0 + \frac{i}{3}kvf_1 - \frac{i}{3}kvu_i\frac{\partial F_0}{\partial v} = C_{ee}[f_0], \quad (1)$$

$$ikvf_0 + i\frac{e}{m_e}k\phi\frac{\partial F_0}{\partial v} - (\tilde{\nu}_1 - \nu_{ei})u_i\frac{\partial F_0}{\partial v} = -\tilde{\nu}_1 f_1. \quad (2)$$

The e-e collision integral C_{ee} in Eq. (1) has been lin-

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earized with respect to the electron Maxwellian distribution function $F_0(v)$. Equations (1) and (2) are written in the ion reference frame and contain an IAW potential ϕ and an ion velocity u_i which satisfy the standard set of cold ion fluid equations [1]. From the ion fluid equations and Eq. (2) one can find the relation between the first angular harmonic $f_1(v) = -i(kv/\tilde{\nu}_1)\psi_1(v)F_0(v)e\phi/T_e$ and the symmetric part of the electron distribution $f_0(v) = [1+\psi_0(v)]F_0(v)e\phi/T_e$,

$$\psi_1 = \psi_0 - i(\omega\nu_{ei}/k^2v_{T_e}^2)[H_1(kv/\nu_{ei}) - 1](1+J_N), \quad (3)$$

where $J_N = (4\pi/n_e)\int_0^\infty dvv^2\psi_0F_0$ is the integral related to the electron density perturbation $\delta n_e/n_e = (1+J_N)e\phi/T_e$. The ion acoustic dispersion relation which follows from these equations, has the following form:

$$\omega^2 = k^2c_s^2(1+J_R)/(1+J_N), \quad (4)$$

where $c_s = \sqrt{ZT_e/m_i}$ and

$$J_R = (4\pi/3n_ev_{T_e}^2)\int_0^\infty dvv^4H_1^{-1}\psi_1F_0$$

is the integral related to the friction force. Substituting Eq. (3) into Eq. (1) we can derive equation for the symmetric part of the electron distribution function

$$\left(\frac{k^2v^2}{3\tilde{\nu}_1} - i\omega\right)\psi_0 = i\omega\left(1 - \frac{v^2}{3v_{T_e}^2}\frac{1+J_N}{H_1}\right) + F_0^{-1}C_{ee}[\psi_0F_0]. \quad (5)$$

The e-e collision operator C_{ee} is approximated by the standard Landau or Rosenbluth potential form (cf., e.g., Ref. [8])

$$C_{ee}[\psi_0F_0] = \frac{1}{Z}\nu_{ei}(v)v\frac{d}{dv}\left(F_0(v)G\left[\frac{d\psi_0}{vdv}\right]\right), \quad (6)$$

where the functional $G[h(v)]$ reads

$$G[h] = \frac{4\pi}{n_0}\left(v_{T_e}^2h(v)\int_0^v dw\, w^2F_0(w) - \frac{1}{3}\int_0^v dw\, w^4F_0(w)h(w) - \frac{v^3}{3}\int_v^\infty dw\, wF_0(w)h(w)\right).$$

Functions $\psi_0 = \text{const}$ and $\psi_0 \propto v^2$ are invariants of this collisional operator. They correspond to electron number density and energy conservation. The new results of our study will follow from the approximate solution of Eq. (5).

Comparing magnitudes of the different terms in Eq. (5) one can see that for thermal electrons, $v \sim v_{T_e}$, the e-e collision integral is small if $k\lambda_{ei} > Z^{-1/2}$. The second term in parentheses on the left hand side is also small for $k\lambda_{ei} > c_s/v_{T_e}$ and therefore in this region of wavelengths Eq. (5) gives an approximate solution for the function $\psi_0 \approx \psi_{0T}$ in the regime of thermal electron velocities

$$\psi_{0T} = i\frac{\omega\nu_{ei}}{k^2v_{T_e}^2}\left[3\frac{v_{T_e}^2}{v^2}H_1\left(\frac{kv}{\nu_{ei}}\right) - 1\right], \quad (7)$$

where we have also neglected the small J_N correction. The function ψ_{0T} (7) is imaginary and of absolute value much less than one. It mainly contributes to the damping of IAW in the dispersion relation (4) through the integrals J_N and J_R. Unfortunately ψ_{0T} diverges as v^{-5} for small velocities and a more accurate solution of Eq. (5) has to be found in order to obtain finite values for J_N and J_R. The e-e collision term in Eq. (5) dominates in the small velocity regime, i.e., for $v < v_* \equiv v_{T_e}(9\sqrt{\pi/2}/Zk^2\lambda_{ei}^2)^{1/7}$. The small imaginary term on the left hand side of Eq. (5) which is related to the nonstationary electron response can also play an important role but only for unrealistically high $Z > v_{T_e}/c_s$ [3].

In order to find ψ_0 which is valid at $v \sim 0$, we keep dominant terms with respect to the small parameter v_*/v_{T_e} in Eq. (5) and derive an approximate equation for ψ_0 for $v \lesssim v_* \ll v_{T_e}(k\lambda_{ei})^{-1/4}$,

$$\frac{k^2v^2}{3\nu_{ei}(v)}\psi_0 = i\omega + \frac{1}{3Z}\sqrt{\frac{2}{\pi}}\nu_{ei}(v)\frac{v}{v_{T_e}}\frac{d}{dv}\left(v^2\frac{d\psi_0}{dv}\right). \quad (8)$$

A similar equation has been derived by Maximov and Silin [9] in the context of laser beam thermal filamentation instability and then extended for the ion acoustic wave in Ref. [10]. The solution of Eq. (8) can be written in terms of a modified Bessel function of the 1/7th order [10]. It is more convenient in practice, however, to use an approximate expression for ψ_0, particularly in the calculation of moments like J_N and J_R,

$$\psi_0(v) \approx iZ\frac{\omega\lambda_{ei}}{v_{T_e}}\left(\frac{v_*}{v_{T_e}}\right)^2\frac{c_\psi}{1+c_\psi(v/v_*)^5}, \quad (9)$$

where $c_\psi = 0.432$ has been found numerically. Equation (9) fits the numerical solution with an accuracy better than 20%. Our approximate procedure of finding ψ_0 is also justified by a limited validity of an asymptotic expansion in v_*/v_{T_e} which has produced Eq. (8). The parameter of this expansion, v_*/v_{T_e}, is close to unity for the interesting range of parameters because of the small 1/7 power in its definition, thus restricting accuracy of Eq. (8). Combining the small velocity asymptotic expression (9) with the solution (7) which is valid for $v \gtrsim v_*$ we can write an approximate distribution function in the following form: $\psi_0(v) = c_\psi v^5\psi_{0T}/(c_\psi v^5 + v_*^5)$.

The dispersion equation (4) in the weakly collisional regime $k\lambda_{ei} \gg c_s/v_{T_e}, Z^{-1/2}$ has the solution $\omega = kc_s - i\gamma$, where $\gamma/kc_s = \frac{1}{2}\,\mathrm{Im}\,(J_N - J_R)$. In the collisionless limit we can evaluate γ by letting $\nu_{ei} \to 0$ in Eq. (7). This results in $J_R \to 0$, $J_N \to i\sqrt{\pi/2}(\omega/kv_{T_e})$, and in the well-known expression for collisionless electron Landau damping. The contribution to J_N from the region of small velocities $v \sim v_*$, where the distribution (9) is valid, scales as $\psi_0(0)(v_*/v_{T_e})^3$ in the case of finite collision frequency. The contribution to γ/kc_s from J_R is $(v_*/v_{T_e})^2$ times smaller than contribution from J_N and can be neglected. Therefore we can write the following expression for the electron part of IAW damping:

$$\frac{\gamma}{kc_s} = \sqrt{\frac{\pi}{8}}\frac{c_s}{v_{T_e}}\left[1 + c_\gamma Z^{2/7}(k\lambda_{ei})^{-3/7}\right], \quad (10)$$

where the numerical integration of ψ_0 in J_N gives $c_\gamma = 1.2$ when $v_*/v_{T_e} \ll 1$. This expression (10) coincides exactly with Eq. (3) derived independently in Ref. [11]. Comparison with the Fokker-Planck simulation results [1] has shown that Eq. (10) overestimates the effect of e-e

collisions by a factor of about 2. The same comparison, however, confirmed the correct scaling of (10) with $k\lambda_{ei}$. We can easily improve the agreement between Eq. (10) and results of Fokker-Planck simulations [1] by introducing a better approximation to ψ_0 (9) and subsequently changing the value of constant c_γ to 0.6. Figure 1 shows good agreement with kinetic suimulations in the region $k\lambda_{ei} \gtrsim 1$.

So far we have discussed high ionization states, $Z \gg 1$. For comparison with data for arbitrary Z we have followed the prescription of Epperlein [1] and modified the electron mean free path, $\lambda^*_{ei} = \lambda_{ei}\zeta(Z)/\zeta(\infty)$ where $\zeta(Z)/\zeta(\infty) = (Z+0.24)/(Z+2.4)$ is the ratio of charge-dependent coefficients in Braginskii's [2] electron thermal conductivity $\kappa_0 = \zeta(Z)n_e v_{T_e}\lambda_{ei}$. This substitution leads to reasonable agreement with the Fokker-Planck simulations even for $Z = 1$ (cf. Fig. 1). Equation (10) shows that the contribution of e-e collisions to IAW damping becomes significant for very short wavelengths $k\lambda^*_{ei} \sim 0.3Z^{2/3}$. Electrons with small velocities $v \sim v_*$ are the main contributors to these new terms in Eq. (10).

After studying solutions to Eq. (5) in collisionless and weakly collisional limits we will construct the electron distribution function in the hydrodynamical regime of $k\lambda^*_{ei} \lesssim Z^{-1/2}$. The e-e collision term now dominates in Eq. (5) and it can no longer be treated as a small correction. Our method of solution of Eq. (5) parallels the classical approach of the Chapman-Enskog theory in which the zero order approximation to the distribution function is constructed from collisional invariants and takes the form of a linearized local Maxwellian distribution function, $\psi^{(0)}_{0H} = c_0 + c_1 v^2/2v^2_{T_e}$. The coefficients $c_0 = -4c_1$ and $c_1 = 1/(1+ir)$, where $r = (128/3\pi)k^2 v_{T_e}\lambda^*_{ei}/\omega$ are found from the relevant equations of particle and energy conservation, which can be obtained by integrating Eq. (5) with v^2F_0 and v^4F_0. The zero order approximation to the electron distribution function $\psi^{(0)}_{0H}$ leads to the well-known dispersion relation for hydrodynamic IAW: $\omega^2/k^2c_s^2 = 1 + 2/(3+2ir)$. Fokker-Planck simulations [1] have demonstrated that the deviation from a hydrodynamical approximation occurs when $k\lambda^*_{ei} > c_s/v_{T_e}$ or $r \gg 1$. We improve the dispersion relation for the IAW by finding the first order correction $\psi^{(1)}_{0H}$ to the electron distribution function. We use $Z(k\lambda^*_{ei})^2$ as a small parameter and solve Eq. (5) for $\psi^{(1)}_{0H}$. Similarily to the standard Chapman-Enskog method this solution involves an inversion of e-e collision operator. The improved expression for the IAW damping reads as

$$\frac{\gamma}{kc_s} = \frac{3\pi}{256}\frac{c_s}{v_{T_e}}\frac{1}{k\lambda^*_{ei}}(1 + 264Zk^2\lambda^{*2}_{ei}). \tag{11}$$

The first term in parentheses on the right hand side of Eq. (11) corresponds to classical electron collisional damping, related to e-i collisions. The second term in parentheses accounts for e-e collision contributions. Comparing both terms we can see that deviation from classical hydrodynamical damping occurs at relatively small wave numbers $k\lambda^*_{ei} \sim 0.06Z^{-1/2}$ because of the large coefficient in front of our small expansion parameter $Z(k\lambda^*_{ei})^2$. This is consistent also with the simulation results (cf. Fig. 1 in Ref. [1]). The region of applicability of Eq. (11) is restricted to very small wave numbers, but we can construct an approximate formula for IAW damping which extends into the whole intermediate region of collisionalities, if we combine Eqs. (10) and (11) in the following expression:

$$\frac{\gamma}{kc_s} = \sqrt{\frac{\pi}{8}}\frac{c_s}{v_{T_e}} \times \left[1 + \frac{c_\gamma}{k\lambda^*_{ei}}\frac{1 + 264Zk^2\lambda^{*2}_{ei}}{128c_\gamma/3\sqrt{2\pi} + 264(Zk^2\lambda^{*2}_{ei})^{5/7}}\right]. \tag{12}$$

Comparison between this formula and the Fokker-Planck simulations shown in Fig. 1 demonstrates good agreement with all simulation points. This has been achieved with only one adjustable parameter $c_\gamma = 0.6$. The strongest deviation is of the order of a factor less than 2 and occurs for $k\lambda^*_{ei} \lesssim 1$ and $Z \gtrsim 1$, i.e., in the intermediate regime of collisionality and for low Z materials.

The perturbations of the electron distribution function ψ_0 (7), (9), and ψ_{0H} allows us to investigate other plasma properties related to IAW propagation. We derive here an expression for the electron heat conductivity κ_e, which has been a frequently debated subject over the last few years [1,3,12]. In particular, it has been shown that the nonlocal part of κ_e in the weakly collisonal and collisionless regime depends on physical processes which characterize deviations from a plasma reference state. We have found similar properties in our expression for electron heat flux q_e, which have shown several features unique to the physical context of IAW propagation. The electron heat flux associated with IAW $\delta\mathbf{q_e} = (m_e/2)\int d\mathbf{v}\, v^2\mathbf{v}\delta f_e$ is related to the first harmonic of the electron distribution function ψ_1 (3) in the reference frame moving with the ion hydrodynamical velocity u_i, which is equal to the electron average velocity due to charge neutrality. Using equations (3) and (5) we obtain the following expression for the electron heat flux $q_e = -n_eT_eu_i\left(1 + \frac{5}{2}J_N - \frac{3}{2}J_T\right)$, where $J_T = (4\pi/3n_ev^2_{T_e})\int_0^\infty dv\; v^4\psi_0F_0$. In the collisionless regime, i.e., for isothermal IAW, the heat flux becomes $q_e \approx -n_eT_eu_i$. Therefore, in this limit the diffusive part of the total electron energy flux cancels the contribution from the convective part $n_eT_eu_i$, which corresponds to zero net energy transport by isothermal IAW. Also in the weakly collisional regime, where κ_e

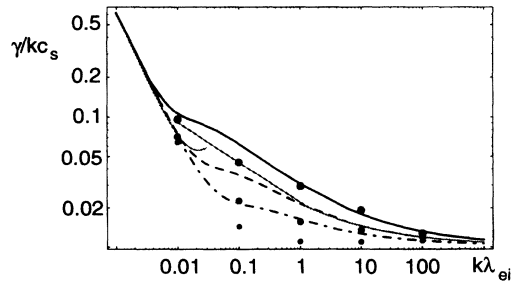

FIG. 1. Comparison of the analytical expression (12) for IAW damping with the results of Fokker-Planck simulations from Ref. [1]. Dot-dashed, dashed, and solid lines correspond to Z =1, 8, and 64, respectively. Asymptotic expressions (light lines) are given for the Z=8 case. Fokker-Planck results are shown as small, medium and large dots for Z = 1, 8, and 64, respectively.

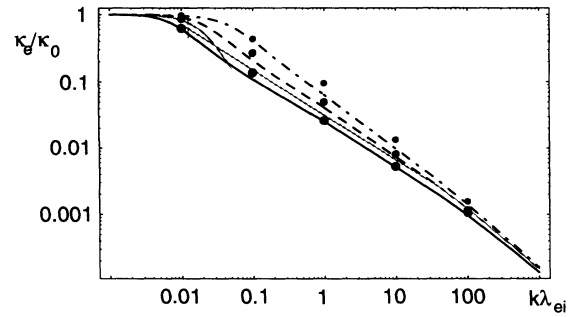

FIG. 2. Comparison of the analytical expression (14) for the electron heat conductivity coefficient with the results of Fokker-Planck simulations from Ref. [1]. Dot-dashed, dashed, and solid lines correspond to Z =1, 8, and 64, respectively. Asymptotic expressions (light lines) are given for the Z=8 case. Fokker-Planck results are shown as small, medium and large dots for Z = 1, 8, and 64, respectively.

usually displays nonlocal behavior, the integrals J_N and J_T are small and the diffusive heat flux q_e does not exhibit any wavelength dependence. These expressions for the electron heat flux are consistent with results of Refs. [3,6], where they also do not contain any collisional effects. The nonlocality of κ_e is therefore a consequence of the wave number dependence of the temperature perturbation $\delta T_e = (4\pi m_e/3n_e)\int_0^\infty dv\ v^2(v^2 - 3v_{T_e}^2)\delta f_0$. Defining the small scale electron heat conductivity κ_e as $q_e/(-ik\delta T_e)$ we obtain the following expression:

$$\frac{\kappa_e}{\kappa_0} = \frac{9\sqrt{2\pi}}{128}\frac{1}{k\lambda_{ei}^* + 3c_\gamma Z^{2/7}(k\lambda_{ei}^*)^{4/7}} \qquad (13)$$

which is valid in the weakly collisional limit $k\lambda_{ei}^* \gtrsim 1$ and the coefficient $c_\gamma = 0.6$ is the same as in Eqs. (10) and (12). The form of wavelength dependence in Eq. (13) is very similar to that of an IAW damping (10), with the exception of the coefficient in front of the second term in the denominator. This term is three times larger than the corresponding factor in Eq. (10), which results in a deviation of κ_e (13) from the collisionless Hammett and Perkins [4] limit as early as $k\lambda_{ei}^* \sim 20Z^{2/3}$. In the weakly collisional limit the electron heat conductivity coefficient (13) exhibits a weak dependence on the IAW wave number $\kappa_e \propto (k\lambda_{ei}^*)^{-4/7}$. This dependence of the electron heat conductivity is very different from those previously reported for the case of the bremsstrahlung heating [9,12,13], $(k\lambda_{ei}^*)^{-10/7}$, or $(k\lambda_{ei}^*)^{-4/3}$. It is also quite different from predictions of Ref. [11] for the ion acoustic wave case.

In the collision-dominated regime the electron distribution function is given by ψ_{0H} as was described above during derivation of Eq. (12). It leads to a k^2 dependence of the electron heat conductivity (cf. Ref. [14]) $\kappa_e/\kappa_0 = 1/(1 + 264Zk^2\lambda_{ei}^{*2})$. This equation is only valid when the second term in the denominator is small. In fact, this expression defines the point of departure of the electron heat flux from the classical Braginskii expression κ_0. Combining our results from the collision dominated regime and (13) in one approximate expression

$$\frac{\kappa_e}{\kappa_0} = \frac{9\sqrt{2\pi}}{128}\left[k\lambda_{ei}^* + \frac{3c_\gamma(1 + 264Zk^2\lambda_{ei}^{*2})}{128c_\gamma/3\sqrt{2\pi} + 264(Zk^2\lambda_{ei}^{*2})^{5/7}}\right]^{-1}, \qquad (14)$$

we are able to reproduce Fokker-Planck simulation results with a good accuracy (cf. Fig. 2) in the entire region of collisionality [15].

In summary, we have constructed the solution of the kinetic equation for the electron distribution function disturbed by an IAW in plasmas with arbitrary collisionality parameter. We have calculated analytically expressions for the ion acoustic damping and the electron heat conductivity. It is found that the *e-e* collisions modify the imaginary part of an electron distribution function and have a strong effect on the IAW damping and the electron heat conductivity.

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[15] In fact the electron heat conductivity is a complex quantity [1]. We discuss here only the absolute value of κ_e. Its imaginary part also follows from Eq. (5) and agrees well with simulations of Ref. [1]. We defer the discussion of this issue for a future publication.